

Convexity is an approach to equity tail risk hedging that is designed to generate gains from index options during extreme volatility events. The approach gets its name from the shape of the payoff diagram of a typical index option contract used in such a strategy. An example of such a payoff diagram is shown as the dark blue line in Figure 1.

Convexity strategies generally focus on purchasing a relatively small amount of an asset or derivative that has the potential for a multifold increase in price during a catastrophic market event. A key takeaway from Figure 1 is that for a convexity hedge to produce a large payoff, the value of the equity index needs to fall both far and fast. If the decline of the equity index value is shallow or slow, the hedge may not experience a significant increase in value and may lose value over time.

The Black-Scholes option pricing model used in Figure 1 assumes an instantaneous change in equity index value. While this extreme assumption may be unrealistic, the shape of the dark blue line will be similar if the change in index value change takes place over a day, a week or even several weeks.

To illustrate this, we can look at the actual historical price change of an equity index put option with similar parameters to the one modeled in Figure 1.

The price of the index put option featured in Figure 2 increased by more than 1,800%. The price increase materialized suddenly in early March 2020 as the equity market plummeted in response to COVID. This dramatic price increase is convexity in action. Note that the option's price moves in both directions along the convexity curve—after spiking to its peak the option's price drops just as dramatically on days the equity market rises. The "double-edged sword" nature of convexity instruments necessitates an active and thoughtful approach to monetization so that hedging gains can be preserved when they materialize.

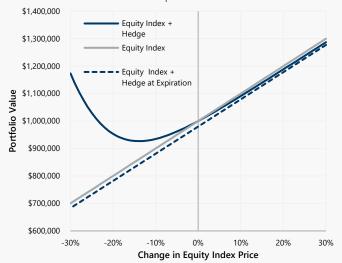
Also note that in the three months prior to the COVID Shock, the value of the option in Figure 2 steadily declined as the price of the S&P 500® Index increased. This is the expected price change as illustrated in the right half of Figure 1, and the longer-term impact of this feature of convexity is illustrated in Figure 3.

## (continued on the next page)

The payoff diagram in Figure 1 is based on a classic Black-Scholes option pricing model, it illustrates the combined value of equity index put options (the hedge) and a portfolio of stocks that replicate an equity index (the equity index) across a range of price changes for the equity index. Specifically, the starting value of the combination is \$1 million, with \$990,000 allocated to the equity index and \$10,000, or 1% of the portfolio value, allocated to the hedge. The index put options have a strike price that is 30% below the index price, i.e. 30% out-of-the-money (OTM), and the put options' expiration date is four months away.

## Figure 1 - Convexity Pricing Model: 1% Allocation to 4-Month, 30% OTM Index Puts

Portfolio Value Under Assumption of Instantaneous Change in Equity Index Value vs. Portfolio Value at Expiration



The dark blue line in Figure 1 shows:

- If the equity index loses enough value fast enough, the increase in value of the hedge can more than offset the loss of the equity index.
- At an index loss of 24% or more, the value of the hedge has increased enough to more than offset the loss of the equity index
- At an index loss of 30%, the value of the hedge has increased to more than \$480,000—a return of more than 4,700% from the starting value of \$10,000!
- At index losses of less than 24%, the increase in the value of the hedge has mitigated the losses of the equity index
- When the value of the equity index increases, the value of the hedge falls and lowers the overall value of the convexity strategy.

The dashed dark blue line in Figure 1 shows the combined value of the equity index and the hedge at the expiration date of the index put options, across a range of price changes for the equity index.

The dashed dark blue line shows that if the index increases in value, the index put options expire worthless and the combined value of equity index and the hedge is \$10,000 less than the outcome for \$1,000,000 invested in only the equity index

Similarly, because the strike price of the index put options is 30% below the starting price of the equity index, the hedge will expire worthless if the index is not more than 30% below its starting price on the index options' expiration date.

Figure 2 - 30% OTM S&P 500<sup>®</sup> Index Put Option vs. S&P 500<sup>®</sup> Index\*

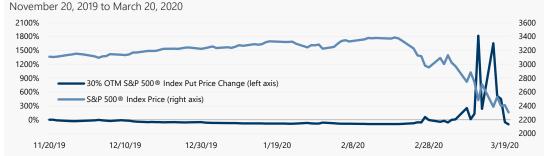


Figure 2 shows the price change in percentage terms of an S&P 500° Index put option with a strike price of 2175 and an expiration date of March 20, 2020. This put option was approximately 30% out-of-the money on November 20, 2019.

As the S&P 500° Index advanced 9.45% from November 20, 2019 through February 19, 2020 option lost over 90% of its value. During the

fast and deep decline of the S&P 500° Index during Q1 of 2020, this index option increased to a peak value more than 1,800% greater than its starting value. The option's price was also extremely sensitive to changes in S&P 500° Index price as the market gyrated in mid-March 2020. Ultimately, the option lost all of its value by the time it expired, as the price of the S&P 500° Index had not fallen below 2175.



Moving beyond the dynamics of a single option, Figure 3 illustrates the impact of a convexity strategy that purchases a portfolio of index put options designed to deliver convex payoffs during equity market catastrophes.

Because crises and catastrophic market events are relatively rare, convexity strategies, like many approaches to tail risk hedging, have a negative expected return. In this way, they are like some forms of insurance—buyers of property insurance do not expect a positive long-term return on the regular premiums they pay, but they do expect a big payoff in the event of a catastrophe that destroys the insured property. Similarly, there may be a long-term cost to a convexity strategy but 1) the expectation of a significant payoff during a market catastrophe is reasonable 2) the cost of protection can be set in advance and budgeted for and 3) the cost of protection is potentially lower than the cost of diversification, particularly in a low- to risinginterest rate environment that may produce low returns from fixed income investments.

The negative expected return of convexity strategies requires thoughtful strategy design that is tailored to a specific investor's objective, risk tolerance, and budget. Moreover, the unique market response to a particular crisis reduces the likelihood that systematic, or rules-based, approaches will achieve the same level of effectiveness from crisis to crisis. Comparing the returns of a simple, rules-based hypothetical convexity strategy during the Great Financial Crisis and the COVID Shock illustrates that the outcome of the same approach can be meaningfully different even over periods that feature similarly extreme market losses and implied volatility levels. Figures 4 and 5 also show that effective monetization to preserve hedging gains can make a significant difference. But, as with the basic strategy parameters, the same approach to monetization can have dramatically different outcomes in different scenarios.

In light of these insights and others, Gateway's framework for customizing convexity strategies to specific investor objectives applies the following:

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Do Not:	Do:
Assume systematic strategies that worked in a past crisis will be similarly successful in the future	Apply quantitative insights about option dynamics to an active decision-making framework
Take basis risk to access low-cost protection	Prioritize low-basis protection
Match the tenor of protection to the investor's time horizon	Ladder shorter-term tenors over a longer-term horizon
Match the strike price of protection to the investor's loss tolerance	Focus on convexity potential, set a budget and spend within it
Limit the notional value of protection to the asset value of the equity portfolio	Focus on notional leverage
Hold protection to expiration when it has appreciated	Take an "early and incremental" approach to monetizing protection





This multi-year time period does not include any crises or equity bear markets, making it useful for comparing the cost of lowering risk with diversification to the cost of a relatively simple convexity-based approach to tail risk hedging. The Diversified Stock and Bond Portfolio shown is 60% S&P 500° Index and 40% Bloomberg U.S. Aggregate Bond Index, rebalanced monthly. The Hypothetical SPX Convexity Strategy shown owns the S&P 500° Index and spends 25 basis points (bps) of equity portfolio value each month (3% per year) on

S&P 500° Index put options. At the time of purchase, the options are three months from expiration and 30% out-of-the-money. The options are held until expiration. Over this time period, all options purchased expired worthless as there were no significant downside events. While catastrophic protection was not needed over this time period, and purchasing it detracted significantly from return, the cost of protection was lower than the cost of traditional diversification.

Figure 4 - Cumulative Appreciation (Starting Value = 100)\*\*



Figure 5 - Cumulative Appreciation (Starting Value = 100)\*\*

December 31, 2019 to June 30, 2020

220 —— S&P 500® Index

190 —— Hypothetical SPX Convexity Strategy

160 —— Hypothetical SPX Convexity Strategy with
VIX®-Based Monetization Threshold

130

100

70

40

12/31/19 2/4/20 3/10/20 4/14/20 5/19/20 6/23/20

Figures 4 & 5 compare two versions of the Hypothetical SPX Convexity Strategy during the Great Financial Crisis and the COVID Shock. The dark blue line in Figures 4 & 5 is the convexity strategy as described in Figure 3. The light grey line strategy buys the same options but sells

them when the Cboe® Volatility Index (the VIX®) Index reaches 60. Note both approaches provided protection in both crises, but monetization made a much bigger difference in the COVID Shock even though VIX® exceeded 60 in both crises.

## IMPORTANT INFORMATION

Performance data shown represents hypothetical performance and is no guarantee of, and not necessarily indicative of, future results. The performance numbers presented above do not represent the results of actual trading. Gateway Investment Advisers, LLC did not provide the advisory service depicted above during the performance period referenced. Data sources: Bloomberg, L.P., Gateway Investment Advisers, LLC and Volos Software. \*\*All hypothetical strategies are for illustrative purposes only and are not available for investment nor actual strategies presented do not reflect advisory fees charged. Diversified Stock and Bond Portfolio was designed by Volos Software and is not available for investment nor an actual strategy managed by Gateway.

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